NAG Toolbox for MATLAB

f01rj

1 Purpose

f01rj finds the RQ factorization of the complex m by n ($m \leq n$), matrix A, so that A is reduced to upper triangular form by means of unitary transformations from the right.

2 Syntax

```
[a, theta, ifail] = f01rj(a, 'm', m, 'n', n)
```

3 Description

The m by n matrix A is factorized as

$$A = (R \quad 0)P^{\mathrm{H}} \quad \text{when } m < n,$$

$$A = RP^{\mathrm{H}} \quad \text{when } m = n,$$

where P is an n by n unitary matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the $(m - k + 1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = I - \gamma_k u_k u_k^H,$$

where

$$u_k = \begin{pmatrix} w_k \\ \zeta_k \\ 0 \\ z_k \end{pmatrix}.$$

γ_k is a scalar for which $\mathrm{Re}(\gamma_k) = 1.0$, ζ_k is a real scalar, w_k is a $(k - 1)$ element vector and z_k is an $(n - m)$ element vector. γ_k and u_k are chosen to annihilate the elements in the kth row of A.

The scalar γ_k and the vector u_k are returned in the kth element of **theta** and in the kth row of **a**, such that θ_k, given by

$$\theta_k = (\zeta_k, \mathrm{Im}(\gamma_k)).$$

is in $\mathbf{theta}(k)$, the elements of w_k are in $\mathbf{a}(k,1), \ldots, \mathbf{a}(k,k-1)$ and the elements of z_k are in $\mathbf{a}(k,m+1), \ldots, \mathbf{a}(k,n)$. The elements of R are returned in the upper triangular part of **a**.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H 1965 *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The leading m by n part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m – int32 scalar**

m, the number of rows of the matrix A.

When $\mathbf{m} = 0$ then an immediate return is effected.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by m upper triangular part of **a** will contain the upper triangular matrix R, and the m by m strictly lower triangular part of **a** and the m by $(n - m)$ rectangular part of **a** to the right of the upper triangular part will contain details of the factorization as described in Section 3.

2: **theta($*$) – complex array**

Note: the dimension of the array **theta** must be at least $\max(1, \mathbf{m})$.

theta(k) contains the scalar θ_k for the $(m - k + 1)$th transformation. If $P_k = I$ then $\mathbf{theta}(k) = 0.0$; if

$$T_k = \begin{pmatrix} I & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & I \end{pmatrix}, \qquad \mathrm{Re}(\alpha) < 0.0$$

then $\mathbf{theta}(k) = \alpha$, otherwise **theta**$(k)$ contains θ_k as described in Section 3 and θ_k is always in the range $\left(1.0, \sqrt{2.0}\right)$.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, **m** < 0,
or **n** $<$ **m**,
or **lda** $<$ **m**.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R0)P^{\mathrm{H}} = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see x02aj), c is a modest function of m and n, and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $8m^2(3n - m)/3$.

The first k rows of the unitary matrix P^{H} can be obtained by calling f01rk, which overwrites the k rows of P^{H} on the first k rows of the array **a**. P^{H} is obtained by the call:

```
[a, ifail] = f01qk('Separate', m, k, a, theta);
```

WORK must be a $\max(m - 1, k - m, 1)$ element array. If K is larger than M, then **a** must have been declared to have at least K rows.

9 Example

```
a = [complex(0, -0.5), complex(0.4, -0.3), complex(0.4, +0), complex(0.3,
+0.4), complex(0, +0.3);
         complex(-0.5, -1.5), complex(0.9, -1.3), complex(-0.4, -0.4),
complex(0.1, -0.7), complex(0.3, -0.3);
       complex(-1, -1), complex(0.2, -1.4), complex(1.8, +0), complex(0,
+0), complex(0, -2.4)];
[aOut, theta, ifail] = f01rj(a)
```

```
aOut =
  Columns 1 through 4
    0.7878               -0.2549 - 0.4006i   -0.2774 - 0.2774i   -0.2850 +
0.5586i
    0.0396 + 0.5222i   -2.1122               -1.1094 - 0.5547i    0.1283 +
0.2317i
  -0.2265 + 0.2265i   0.0453 + 0.3171i  -3.6056                 0
  Column 5
   0.1154 + 0.7031i
   0.0790 - 0.0361i
        0 + 0.5436i
theta =
   1.0387 - 0.1006i
   1.1810 + 0.3809i
   1.2244
ifail =
           0
```